Scattering and Depolarization of Electromagnetic Waves by Rough Surfaces

by

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ABSTRACT

A general theory of omnidirectional scattering of electromagnetic waves from rough surfaces is developed under the tangent plane approximation. It is shown that the results obtained are much more general than those of what is usually called the "physical optics" method.

The depolarizing effect of rough surfaces is established, and the reciprocal character of the depolarized component is shown for the back-scattered case.

I. Introduction

Many theories (Hayre, 1961; Daniels, 1963; Fung, 1964; Hagfors, 1964; Beckmann, 1964) on scattering from rough surfaces employ the Kirchhoff-Huygens principle and conclude that there is no depolarization for backscattering. Their results also lead to an expression for backscattered power which is proportional to the square of either one of the Fresnel reflection coefficients, depending on whether the incident plane wave is vertically or horizontally polarized. Such results are actually over-approximated, and consequently the conclusion obtained on depolarization is in error. Such an error shows up also in books on wave propagation and scattering (Beckmann and Spizzichino, 1963; Kerr, 1951). The following discussion will show that depolarization is generally present even in backscattering, and that the return power expression depends on both Fresnel reflection coefficients instead of just the square of one for either vertical or horizontal polarization. Such a result appears to explain the behavior of rough-surface scattering much better than any of the previous theories.

II. The Scattered Field

Consider the case of a plane electromagnetic wave with harmonic time dependence, exp $(j\omega t)$, incident obliquely at an angle, Θ, on a smoothly-undulating, finitely-conducting surface, $z(x,y)$ (see Figure 1). The far-zone re-rediated electric field, $\underset{\sim}{E}_s$, at the point, P, can be expressed (Silver, 1947) as follows:

$$\underset{\sim}{E}_s (P) = K \underset{\sim}{n}_2 \times \int \left[\underset{\sim}{n} \times \underset{\sim}{E} - \eta \underset{\sim}{n}_2 \times (\underset{\sim}{n} \times \underset{\sim}{H})\right] \exp (jk\underset{\sim}{r} \cdot \underset{\sim}{n}_2)\, dS, \qquad (1)$$

where $\underset{\sim}{r}$ is a vector from the origin of the coordinate system to the surface element dS,

$\underset{\sim}{n}_2$ is a unit vector from the origin to the field point in the direction α, ϕ, R is the distance from the origin to the field point,

$\underset{\sim}{n}_1$ is a unit vector in the incident direction which lies in y - z plane,

$\underset{\sim}{E}, \underset{\sim}{H}$ are the total electric and magnetic fields on the surface,

η is the intrinsic impedance of free space,

k is the wave number,

$\underset{\sim}{n}$ is the local unit normal to the surface,

$K = (-jk \exp(-jkR) / 4\pi R)$.

Let us now assume the validity of the tangent plane approximation for the problem and determine the values of $\underset{\sim}{n} \times \underset{\sim}{E}$ and $\underset{\sim}{n} \times \underset{\sim}{H}$ at each point on the surface, z (x,y). This can be done by resolving the incident plane wave into local polarization components parallel and normal to a local plane of incidence and then working out the two components separately. To do so, let the incident electric and magnetic field be defined by

$$\underset{\sim}{E}_i = \underset{\sim}{a} E_o \exp(-jk_1 \cdot r) = \underset{\sim}{a} E; \tag{2}$$

$$\underset{\sim}{H}_i = \frac{\underset{\sim}{n}_1 \times \underset{\sim}{a}}{\eta} E; \tag{3}$$

where $E = E_o \exp(-j\underset{\sim}{k}_1 \cdot \underset{\sim}{r})$ and $\underset{\sim}{a}$ is the polarization vector; and $\underset{\sim}{k}_1 = k\,\underset{\sim}{n}_1$. In view of the local coordinates defined in Figure 2, we can write the normally polarized components of the incident fields as

$$\underset{\sim}{E}_1^i = (\underset{\sim}{a} \cdot \underset{\sim}{t}_1)\, \underset{\sim}{t}_1\, E, \tag{4}$$

$$\underset{\sim}{H}_1^i = (\underset{\sim}{a} \cdot \underset{\sim}{t}_1)\, \underset{\sim}{n}_1 \times \underset{\sim}{t}_1\, E/\eta, \tag{5}$$

and the components polarized in the plane of incidence as

$$\underset{\sim}{E}_2^i = \left[(\underset{\sim}{a} \cdot \underset{\sim}{n})\, \underset{\sim}{n} + (\underset{\sim}{a} \cdot \underset{\sim}{p})\, \underset{\sim}{p} \right] E, \tag{6}$$

$$\underset{\sim}{H}_2^i = \left[(\underset{\sim}{a} \cdot \underset{\sim}{n})\, (\underset{\sim}{n}_1 \times \underset{\sim}{n}) + (\underset{\sim}{a} \cdot \underset{\sim}{p})\, (\underset{\sim}{n}_1 \times \underset{\sim}{p}) \right] E/\eta . \tag{7}$$

From the equations above, we obtain, in view of the boundary conditions from electromagnetic theory (Stratton, 1941),

$$\begin{aligned}\mathbf{n} \times \mathbf{E} &= \mathbf{n} \times \mathbf{E}_1 + \mathbf{n} \times \mathbf{E}_2 \\ &= \mathbf{n} \times \mathbf{E}_1^i\ (1 + R_1) + \mathbf{n} \times \mathbf{E}_2^i + \mathbf{n} \times \mathbf{E}_2^r \\ &= \mathbf{n} \times \mathbf{E}_1^i\ (1 + R_1) + \mathbf{n} \times (\mathbf{H}_2^i \times \mathbf{n}_1)\ \eta + \mathbf{n} \times (\mathbf{H}_2^i\ R_2 \times \mathbf{n}_o)\eta \\ &= \mathbf{n} \times \mathbf{E}_1^i\ (1 + R_1) + (\mathbf{n} \cdot \mathbf{n}_1)\ \eta \mathbf{H}_2^i\ (1 - R_2),\end{aligned} \tag{8}$$

where $\mathbf{E}_2^r$ is the reflected electric field due to $\mathbf{E}_2^i$,

R_1, R_2 are the Fresnel reflection coefficients for the normally polarized and parallel polarized waves respectively,

n_o is defined in Figure 2.

Similarly, we obtain

$$\begin{aligned}\mathbf{n} \times \mathbf{H} &= \mathbf{n} \times \mathbf{H}_1 + \mathbf{n} \times \mathbf{H}_2 \\ &= (\mathbf{n} \cdot \mathbf{n}_1)(1 - R_1)\ \mathbf{E}_1^i/\eta + \mathbf{n} \times \mathbf{H}_2^i\ (1 + R_2),\end{aligned} \tag{9}$$

where we apply the fact that $\mathbf{n} \cdot \mathbf{n}_1 = \mathbf{n} \cdot \mathbf{n}_o$ in (8) and (9).
To obtain $\mathbf{n} \times \mathbf{E}$ and $\mathbf{n} \times \mathbf{H}$ in terms only of the incident electric field, we can substitute (4) and (7) in (8) and (9). These give

$$\begin{aligned}\mathbf{n} \times \mathbf{E} &= (1 + R_1)\ \mathbf{p}\ (\mathbf{a} \cdot \mathbf{t}_1)\ E \\ &\quad + (\mathbf{n} \cdot \mathbf{n}_1)(1 - R_2)\left[(\mathbf{a} \cdot \mathbf{n})\ (\mathbf{n}_1 \times \mathbf{n}) + (\mathbf{a} \cdot \mathbf{p})\ (\mathbf{n}_1 \times \mathbf{p})\right] E \\ &= (1 + R_1)\ (\mathbf{a} \cdot \mathbf{t}_1)\ \mathbf{p}\ E \\ &\quad - (\mathbf{n} \cdot \mathbf{n}_1)\ (1 - R_2)\ \left[(\mathbf{a} \cdot \mathbf{n})(\mathbf{n} \cdot \mathbf{d}_1)\ \mathbf{t}_1 + (\mathbf{a} \cdot \mathbf{p})\ (\mathbf{p} \cdot \mathbf{d}_1)\ \mathbf{t}_1\right] E;\end{aligned} \tag{10}$$

$$\begin{aligned}\mathbf{n} \times \mathbf{H} &= -\ (\mathbf{n} \cdot \mathbf{n}_1)\ (1 - R_1)\ (\mathbf{a} \cdot \mathbf{t}_1)\ \mathbf{t}_1\ E/\eta \\ &\quad - (1 + R_2)\ \left[(\mathbf{a} \cdot \mathbf{n})\ (\mathbf{n} \cdot \mathbf{d}_1)\ \mathbf{p} + (\mathbf{a} \cdot \mathbf{p})\ (\mathbf{p} \cdot \mathbf{d}_1)\ \mathbf{p}\right] E/\eta\,.\end{aligned} \tag{11}$$

Substituting (10) and (11) into (1), we obtain

$$\underset{\sim}{E}_s\,(P) = K\,\underset{\sim}{n}_2 \times \int E \Big\{ (1+R_1)\,(\underset{\sim}{a}\cdot\underset{\sim}{t}_1)\,\underset{\sim}{p} - (\underset{\sim}{n}\cdot\underset{\sim}{n}_1)\,(1-R_2)$$

$$\Big[(\underset{\sim}{a}\cdot\underset{\sim}{n})\,(\underset{\sim}{n}\cdot\underset{\sim}{d}_1)\,\underset{\sim}{t}_1 + (\underset{\sim}{a}\cdot\underset{\sim}{p})\,(\underset{\sim}{p}\cdot\underset{\sim}{d}_1)\,\underset{\sim}{t}_1\Big] + (\underset{\sim}{n}\cdot\underset{\sim}{n}_1)\,(1-R_1)(\underset{\sim}{a}\cdot\underset{\sim}{t}_1)\,(\underset{\sim}{n}_2 \times \underset{\sim}{t}_1)$$

$$+ (1+R_2)\,\Big[(\underset{\sim}{a}\cdot\underset{\sim}{n})\,(\underset{\sim}{n}\cdot\underset{\sim}{d}_1)\,(\underset{\sim}{n}_2 \times \underset{\sim}{p}) + (\underset{\sim}{a}\cdot\underset{\sim}{p})\,(\underset{\sim}{p}\cdot\underset{\sim}{d}_1)\,(\underset{\sim}{n}_2 \times \underset{\sim}{p})\Big]\Big\}$$

$$\exp\,(j\,k\underset{\sim}{r}\cdot\underset{\sim}{n}_2)\;d\,S$$

$$= K\int E\,\exp\,(jk\underset{\sim}{r}.\underset{\sim}{n}_2)\,\Big\{\Big[\,(1+R_1)\,(\underset{\sim}{a}\,.\,\underset{\sim}{t}_1)\,(\underset{\sim}{p}\,.\,\underset{\sim}{t}_2)$$

$$-\Big[(\underset{\sim}{n}.\underset{\sim}{n}_1)(1-R_2)\,(\underset{\sim}{t}_1\,.\,t_2) + (\underset{\sim}{p}.\underset{\sim}{d}_2)\,(1+R_2)\Big]\Big[\,(\underset{\sim}{a}.\underset{\sim}{n})\,(\underset{\sim}{n}.\underset{\sim}{d}_1) + (\underset{\sim}{a}.\underset{\sim}{p})(\underset{\sim}{p}.\underset{\sim}{d}_1)\Big]$$

$$-\,(\underset{\sim}{n}\,.\,\underset{\sim}{n}_1)\,(1-R_1)\,(\underset{\sim}{a}\,.\,\underset{\sim}{t}_1)\,(\underset{\sim}{t}_1\,.\,\underset{\sim}{d}_2)\,\Big]\,\underset{\sim}{d}_2$$

$$-\,\Big((\underset{\sim}{n}.\underset{\sim}{n}_1)\,(1-R_1)\,(\underset{\sim}{a}.\underset{\sim}{t}_1)\,(\underset{\sim}{t}_1.\underset{\sim}{t}_2) + (\underset{\sim}{a}.\underset{\sim}{t}_1)\,(1+R_1)\,(\underset{\sim}{p}.\underset{\sim}{d}_2)$$

$$+\Big[(\underset{\sim}{p}.\underset{\sim}{t}_2)\,(1+R_2) - (\underset{\sim}{n}.\underset{\sim}{n}_1)\,(1-R_2)\,(\underset{\sim}{t}_1.\underset{\sim}{d}_2)\Big]\Big[\,(\underset{\sim}{a}.\underset{\sim}{n})(\underset{\sim}{n}.\underset{\sim}{d}_1)+(\underset{\sim}{a}.\underset{\sim}{p})(\underset{\sim}{p}.\underset{\sim}{d}_1)\Big]\Big)$$

$$\cdot\,\underset{\sim}{t}_2\Big\}\,dS, \tag{12}$$

where $\underset{\sim}{t}_2$, $\underset{\sim}{d}_2$, are defined in Figure 3.

This is the general result for the scattered field in the direction $\underset{\sim}{n}_2$. Since $\underset{\sim}{d}_2$ and $\underset{\sim}{t}_2$ are vectors with components in x, y, and z directions, and $\underset{\sim}{a}$ is a fixed vector in x direction for horizontal polarization and in y-z plane for vertical polarization, it is seen that depolarization is present in the general case. Another way to see this is to note that $\underset{\sim}{d}_2$ and $\underset{\sim}{t}_2$ are vectors defined with respect to $\underset{\sim}{n}_2$, which may take on any direction; whereas $\underset{\sim}{a}$ is a fixed vector once the incident polarization is defined.

III. The Backscattered Field

An important special case for the scattered field is that of backscattering, i.e., when $\underline{n}_2 = -\underline{n}_1$, $\underline{d}_2 = \underline{d}_1$ and $\underline{t}_2 = -\underline{t}_1$. In this case, (12) reduces to the following expression:

$$\underline{E}_s = -K \int 2E_o (\underline{n} \cdot \underline{n}_1) \exp(-j\, 2\underline{k}_1 \cdot \underline{r}) \left\{ R_2 \left[(\underline{a} \cdot \underline{n})(\underline{n} \cdot \underline{d}_1) + (\underline{a} \cdot \underline{p})(\underline{p} \cdot \underline{d}_1) \right] \underline{d}_1 - (\underline{a} \cdot \underline{t}_1) R_1 \underline{t}_1 \right\} dS . \tag{13}$$

From (13) it is seen that since the local coordinates defined by $\underline{d}_1$, $\underline{t}_1$, and $\underline{n}$ are independent of the direction of polarization, $\underline{a}$, of the incident field, we again conclude that depolarization is generally present. The effect of depolarization will, however, become very small for the special case of vertical incidence (see appendix). In the case of horizontal polarization, depolarization will also vanish for the special case when $\underline{n}$ is in the plane of incidence, since $\underline{a} \cdot \underline{n}$ and $\underline{a} \cdot \underline{p}$ will then be zero, and $\underline{t}_1$ is parallel to x direction. Similarly, there is no depolarization for the vertical case when the local normal, n, is in the plane of incidence, since $\underline{a} \cdot \underline{t}_1$ is zero. However, the last two cases are of interest for surfaces that are rough in only one dimension.

The above discussion shows that the depolarized field component depends strongly on the local normal, $\underline{n}$, which in turn depends on the surface slopes. Hence, for a surface which is uniformly rough in two dimensions, the depolarization component is, in principle, a measure not only of the surface roughness but also of the average surface slope.

The character of scattering by rough surfaces is brought out most clearly by (13), if we note that $\underline{n}$, $\underline{p}$, $\underline{t}_1$ is a set of local coordinates, and $(\underline{a} \cdot \underline{n})$, $(\underline{a} \cdot \underline{p})$ and $(\underline{a} \cdot \underline{t}_1)$ give the relative sizes of the components of the incident field. The amount of depolarization is then seen to be directly related to the sizes of these components.

The power expression given by

$$p = \frac{1}{2\eta} \underline{E}_s \cdot \underline{E}_s^* \tag{14}$$

clearly involves not only one term with the square of one of the Fresnel reflection coefficients, but also another term or terms multipled by the square of the other Fresnel reflection coefficient. This follows from the direct substitution of (13) into (14). In what follows we shall investigate the reciprocal character of the depolarized field component in backscattering.

IV. Case of Horizontal Polarization

Let us now assume that the incident electric field is polarized perpendicular to the plane of incidence, y-z. Let the unit vectors along the x, y, and z axes be $\underset{\sim}{i}$, $\underset{\sim}{j}$, and $\underset{\sim}{k}$. Then, from calculus, the local unit normal to the surface, Z(x,y), can be expressed in terms of the partial derivatives of the surface as

$$\underset{\sim}{n} = (-\underset{\sim}{i}\, Z_x - \underset{\sim}{j}\, Z_y + \underset{\sim}{k})\, (1 + Z_x^2 + Z_y^2)^{-\frac{1}{2}} \,, \tag{15a}$$

and the differential surface element as

$$d\,S = (1 + Z_x^2 + Z_y^2)^{-\frac{1}{2}}\, dx\, dy \,. \tag{15b}$$

Of course, n_1 can also be expressed in terms of the unit vectors, $\underset{\sim}{j}$ and $\underset{\sim}{k}$, and the incident angle, Θ, as

$$\underset{\sim}{n}_1 = \underset{\sim}{j} \sin\Theta + \underset{\sim}{k} \cos\Theta. \tag{16}$$

Now from Figure 2, and with (15) and (16) we can write the unit vectors $\underset{\sim}{t}_1$, $\underset{\sim}{p}$, $\underset{\sim}{d}_1$ in terms of the unit vectors $\underset{\sim}{i}$, $\underset{\sim}{j}$, $\underset{\sim}{k}$ the angle of incidence, Θ, and the partial derivatives of the surface function. Thus,

$$\begin{aligned} \underset{\sim}{t}_1 &= (\underset{\sim}{n}_1 \cdot \underset{\sim}{n})\, (1 - (\underset{\sim}{n}_1 \cdot \underset{\sim}{n})^2)^{-\frac{1}{2}} \\ &= (\underset{\sim}{i}\, (\sin\Theta - Z_y \cos\Theta) + \underset{\sim}{j}\, Z_x \cos\Theta + \underset{\sim}{k}\, Z_x \sin\Theta)/D_1 \,; \end{aligned} \tag{17}$$

$$\underset{\sim}{p} = \underset{\sim}{n} \times \underset{\sim}{t}_1$$

$$= (- \underset{\sim}{i}(Z_x Z_y \sin\Theta + Z_x \cos\Theta) + \underset{\sim}{j} \, (Z_x^2 \sin\Theta + \sin\Theta - Z_y \cos\Theta)$$

$$+ \underset{\sim}{k} \, (Z_y \sin\Theta - Z_y^2 \cos\Theta - Z_x^2 \cos\Theta) \;) / \, (D_1 D_2) \; ; \qquad (18)$$

$$\underset{\sim}{d}_1 = \underset{\sim}{n}_1 \times \underset{\sim}{t}_1$$

$$= (\underset{\sim}{i} \, Z_x + \underset{\sim}{j} \cos\Theta \; (Z_y \cos\Theta - \sin\Theta)$$

$$- \underset{\sim}{k} \sin\Theta \; (\sin\Theta - Z_y \cos\Theta) \;) / D_1 \; ; \qquad (19)$$

where $D_1 = (1 + Z_x^2 + Z_y^2 - (\cos\Theta + Z_y \sin\Theta)^2 \,)^{\frac{1}{2}}$

$$D_2 = (1 + Z_x^2 + Z_y^2)^{\frac{1}{2}} \quad .$$

With (15) through (19), and noting that $\underset{\sim}{a} = \underset{\sim}{i}$, we can calculate the following quantities, namely

$$(\underset{\sim}{a} \cdot \underset{\sim}{t}_1) = (\underset{\sim}{i} \cdot \underset{\sim}{t}_1) = (\sin\Theta - \cos\Theta \, Z_y)/D_1 \, , \qquad (20a)$$

$$(\underset{\sim}{n} \cdot \underset{\sim}{n}_1) = - \; (\sin\Theta \, Z_y + \cos\Theta)/D_2 \; , \qquad (20b)$$

$$(\underset{\sim}{a} \cdot \underset{\sim}{n}) = (\underset{\sim}{i} \cdot \underset{\sim}{n}) = - \, Z_x / D_2 \; , \qquad (20c)$$

$$(\underset{\sim}{n} \cdot \underset{\sim}{d}_1) = -(Z_x^2 + \cos^2\Theta \, Z_y^2 - 2\sin\Theta \cos\Theta \, Z_y + \sin^2\Theta)/(D_1 D_2) \; , \qquad (20d)$$

$$(\underset{\sim}{a} \cdot \underset{\sim}{p}) = (\underset{\sim}{i} \cdot \underset{\sim}{p}) = - \; (\sin\Theta \, Z_y + \cos\Theta) \, Z_x \, / \, (D_1 D_2) \; , \qquad (20e)$$

$$(\underset{\sim}{p} \cdot \underset{\sim}{d}) = (\underset{\sim}{n} \cdot \underset{\sim}{n}_1) \; . \qquad (20f)$$

Substituting (20) in (13) and simplifying, we get

$$E_s(P) = -2K \int E_o (\sin\Theta\, Z_y + \cos\Theta) / D_1$$

$$\left\{ R_1 (\sin\Theta - \cos\Theta\, Z_y)\, t_1 \right.$$

$$\left. - R_2 Z_x d_1 \right\} \exp(-2j k_1 \cdot r)\, dx\, dy$$

$$= 2K E_o \int \exp(-2j k_1 \cdot r)(\sin\Theta\, Z_y + \cos\Theta) / D_1^{\,2}$$

$$\left\{ i \left[R_2 Z_x^2 - R_1 (\sin\Theta - \cos\Theta\, Z_y)^2 \right] \right.$$

$$\left. -(j \cos\Theta + k \sin\Theta) Z_x (\sin\Theta - \cos\Theta\, Z_y)(R_1 + R_2) \right\} dx\, dy, \qquad (21)$$

where $D_1^2 = (\sin\Theta - \cos\Theta\, Z_y)^2 + Z_x^{\,2}$.

It is interesting to note that the i term in (21) will reduce to the results used by previous authors, if Z_x^2 - terms are neglected. Thus, leaving out the depolarized component in (21), the scattered field takes the form

$$E_s(P) = -i\, 2K E_o \int R_1 (\sin\Theta\, Z_y + \cos\Theta) \exp(-2j k_1 \cdot r)\, dx\, dy\ . \qquad (22)$$

This form is seen to check with equation (5) used by Hagfors[10].

V. Case of Vertical Polarization

Here we assume that $a = j \cos\Theta + k \sin\Theta$. In the same fashion used for the horizontally polarized case, the scattered field can be obtained by substituting (20) in (13), except now (20a), (20c) and (20e) take the form

$$(a \cdot t_1) = (j \cos\Theta + k \sin\Theta) \cdot t_1 = Z_x / D_1\ ,$$

$$(a \cdot n) = (\sin\Theta - \cos\Theta\, Z_y) / D_2\ ,$$

$$(a \cdot p) = (\sin\Theta \cos\Theta + (\sin^2\Theta - \cos^2\Theta) Z_y - \sin\Theta \cos\Theta\, Z_y^2)/(D_1 D_2)\ .$$

After some simplification, the final form for the scattered field becomes

$$\underset{\sim}{E}_s(P) = -2\,K\int E_o\,(\sin\Theta\,Z_y + \cos\Theta)\,/\,D_1$$
$$\left[R_2\,(\sin\Theta - \cos\Theta\,Z_y)\,\underset{\sim}{d}_1 + R_1\,Z_x\,\underset{\sim}{t}_1\right]$$
$$\exp\,(-2j\underset{\sim}{k}_1 \cdot \underset{\sim}{r})\,dx\,dy$$
$$= 2\,K\,E_o\int \exp\,(-2j\underset{\sim}{k}_1 \cdot \underset{\sim}{r})\,(\sin\Theta\,Z_y + \cos\Theta)\,/\,D_1{}^2$$
$$\left[(R_2\,(\sin\Theta - \cos\Theta\,Z_y)^2 - R_1\,Z_x^2\,)\,(\underset{\sim}{j}\cos\Theta + \underset{\sim}{k}\sin\Theta)\right.$$
$$\left. + \underset{\sim}{i}\,Z_x\,(\sin\Theta - \cos\Theta\,Z_y)\,(R_1 + R_2)\right]\,dx\,dy\ . \tag{23}$$

The depolarized component, $\underset{\sim}{E}_{2d}$, in this case is seen to be

$$\underset{\sim}{E}_{2d} = \underset{\sim}{i}\,2K\,E_o\int \exp\,(-2j\underset{\sim}{k}_1 \cdot \underset{\sim}{r})\,(\sin\Theta\,Z_y + \cos\Theta)\,/\,D_1{}^2$$
$$Z_x\,(\sin\Theta - \cos\Theta\,Z_y)\,(R_1 + R_2)\,dx\,dy\ . \tag{24}$$

The depolarized component, $\underset{\sim}{E}_{1d}$, for the horizontally polarized case is from (18) :

$$\underset{\sim}{E}_{1d} = -\,(\underset{\sim}{j}\cos\Theta + \underset{\sim}{k}\sin\Theta)\,2K\,E_o\int \exp\,(-2j\underset{\sim}{k}_1 \cdot \underset{\sim}{r})\,(\sin\Theta\,Z_y + \cos\Theta)/D_1^2$$
$$Z_x\,(\sin\Theta - \cos\Theta\,Z_y\,)\,(R_1 + R_2)\,dx\,dy\ . \tag{25}$$

Hence, we conclude that aside from the unit vectors, the magnitude of the backscattered depolarized field component obtained by transmitting a horizontally polarized plane wave is the same as that obtained by transmitting a vertically polarized plane wave, except for a minus sign when the scattering process is considered with respect to the same surface.

VI. Conclusions

In general it is found that there is a depolarizing component even for backscattering. The interrelationship between the backscattered field due to an incident horizontally-polarized wave and that due to a vertically-polarized wave is clearly shown in (13). Such a result then provides better understanding of the depolarization process as well as the scattering process. The error in previous works is seen to result from using an over-approximated surface current which does not sufficiently exhibit the effect of the rough surface. While this error may be small for surfaces with small slopes in average return power consideration, it is a serious one in depolarization considerations.

The reciprocal character of the depolarized electromagnetic field resulting from scattering by a smoothly undulating surface is established for the case of backscattering. This is the case of greatest interest, since many radar experiments [11-15] on the moon and other planets, as well as on natural earth terrains, fall into this category. Although the proof is given for any surface to which the tangent plane approximation is applicable, it seems intuitively plausible that any differentiable surface may have the same character.

APPENDIX : Case of Normal Incidence

In view of Figure 2, we obtain

$$\underset{\sim}{t}_1 = \frac{\underset{\sim}{n}_1 \times \underset{\sim}{n}}{\left[1 - (n_1 \cdot n)^2\right]^{1/2}}$$

$$= \frac{\underset{\sim}{i}\,(\sin\theta - Z_y \cos\theta) + \underset{\sim}{j}\, Z_x \cos\theta + \underset{\sim}{k}\, Z_x \sin\theta}{\left[1 + Z_x^{\,2} + Z_y^{\,2} - (\cos\theta + Z_y \sin\theta)^2\right]^{1/2}}$$

$$p_1 = -\underset{\sim}{i}\,(Z_x Z_y \sin\theta + Z_x \cos\theta) + \underset{\sim}{j}\,(Z_x^{\,2} \sin\theta + \sin\theta$$

$$-Z_y \cos\theta) + \underset{\sim}{k}\,(Z_y \sin\theta - Z_y^{\,2} \cos\theta - Z_x^{\,2} \cos\theta)$$

$$\left[(1 + Z_x^{\,2} + Z_y^{\,2})\,(1 + Z_x^{\,2} + Z_y^{\,2} - (\cos\theta + Z_y \sin\theta)^2)\right]^{-1/2}$$

$$d_1 = \underset{\sim}{i}\, Z_x + \underset{\sim}{j} \cos\theta\,(Z_y \cos\theta - \sin\theta) - \underset{\sim}{k} \sin\theta\,(\sin\theta - Z_y \cos\theta)$$

$$\left[1 + Z_x^{\,2} + Z_y^{\,2} - (\cos\theta + Z_y \sin\theta)^2\right]^{-1/2}$$

where Z_x, Z_y are the partial derivatives of the surface, $Z(x,y)$. For vertical incidence, the above expressions reduce to

$$\underset{\sim}{t}_1 = (-\underset{\sim}{i} Z_y + \underset{\sim}{j} Z_x)\ (Z_x^{\,2} + Z_y^{\,2})^{-1/2}$$

$$\underset{\sim}{p}_1 = \left[-\underset{\sim}{i}\, Z_x - \underset{\sim}{j}\, Z_y - \underset{\sim}{k}\,(Z_x^{\,2} + Z_y^{\,2})\right]$$

$$\left[(1 + Z_x^{\,2} + Z_y^{\,2})\,(Z_x^{\,2} + Z_y^{\,2})\right]^{-1/2}$$

$$\underset{\sim}{d}_1 = \left[\underset{\sim}{i}\, Z_x + \underset{\sim}{j}\, Z_y\right]\ (Z_x^{\,2} + Z_y^{\,2})^{-1/2}$$

Assume horizontal polarization i.e., $\underset{\sim}{a} = \underset{\sim}{i}$, then,

$$\mathbf{a} \cdot \mathbf{n} = \mathbf{i}\,(-\mathbf{i}\,Z_x - \mathbf{j}\,Z_y + \mathbf{k})\left[1 + Z_x^2 + Z_y^2\right]^{-\frac{1}{2}}$$

$$= -Z_x\,(1 + Z_x^2 + Z_y^2)^{-\frac{1}{2}}$$

$$\mathbf{n} \cdot \mathbf{d}_1 = -(Z_x^2 + Z_y^2)^{\frac{1}{2}}\,(1 + Z_x^2 + Z_y^2)^{-\frac{1}{2}}$$

$$\mathbf{a} \cdot \mathbf{p} = -Z_x\left[(1 + Z_x^2 + Z_y^2)\,(Z_x^2 + Z_y^2)\right]^{-\frac{1}{2}}$$

$$\mathbf{p} \cdot \mathbf{d}_1 = -(1 + Z_x^2 + Z_y^2)^{-\frac{1}{2}}$$

$$\mathbf{a} \cdot \mathbf{t}_1 = -Z_y\,(Z_x^2 + Z_y^2)^{-\frac{1}{2}}$$

$$(\mathbf{a} \cdot \mathbf{n})\,(\mathbf{n} \cdot \mathbf{d}_1) + (\mathbf{a} \cdot \mathbf{p})\,(\mathbf{p} \cdot \mathbf{d}_1) = Z_x\,(Z_x^2 + Z_y^2)^{-\frac{1}{2}}$$

Substituting the above result into (13), we obtain

$$E_s = -2K \int E_o\,(\mathbf{n} \cdot \mathbf{n}_1)\,\exp(-j\,2\mathbf{k}_1 \cdot \mathbf{r})$$

$$\left[R_2\,Z_x\,(Z_x^2 + Z_y^2)^{-1}\,(\mathbf{i}\,Z_x + \mathbf{j}\,Z_y)\right.$$

$$\left. + R_1\,Z_y\,(-\mathbf{i}\,Z_y + \mathbf{j}\,Z_x)\,(Z_x^2 + Z_y^2)^{-1}\right] dS$$

$$= -2K \int E_o\,(\mathbf{n} \cdot \mathbf{n}_1)(Z_x^2 + Z_y^2)^{-1} \exp(-j\,2\mathbf{k}_1 \cdot \mathbf{r})$$

$$\mathbf{i}\,(R_2\,Z_x^2 - R_1\,Z_y^2) + \mathbf{j}\,Z_x\,Z_y\,(R_1 + R_2)\Big]\,dS$$

At vertical incidence R_1 tends to the negative of R_2 (since the surface is assumed to have small slopes) and, therefore, the term in $\mathbf{j}$ direction becomes very small. Observe that R_1 is not identical with the negative of R_2 for vertical incidence. Although in this case the angle of incidence is zero, the local angle of incidence, of which R_1 and R_2 are functions, may not be: that is, the normal to the surface is not necessarily in $\mathbf{k}$ direction.

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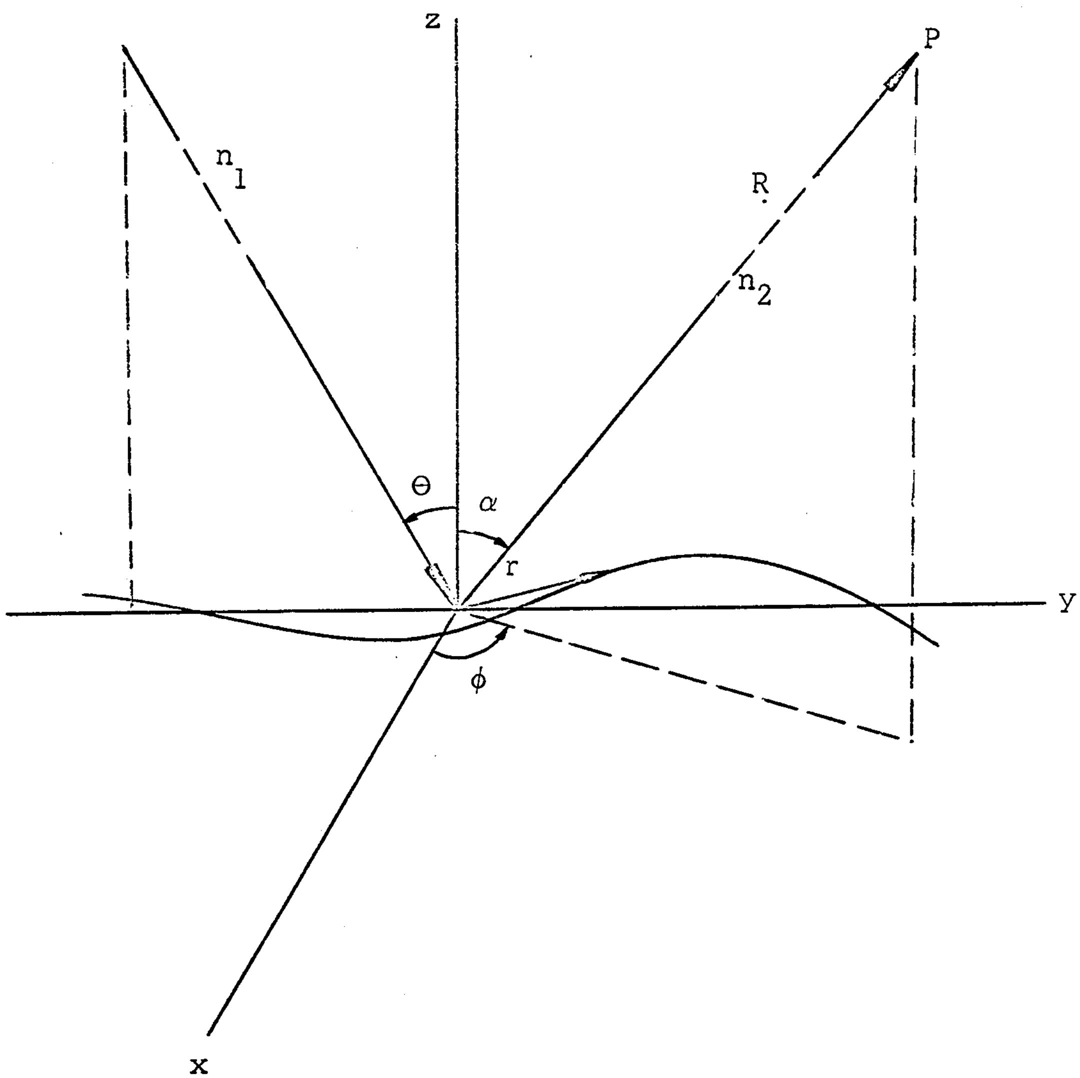
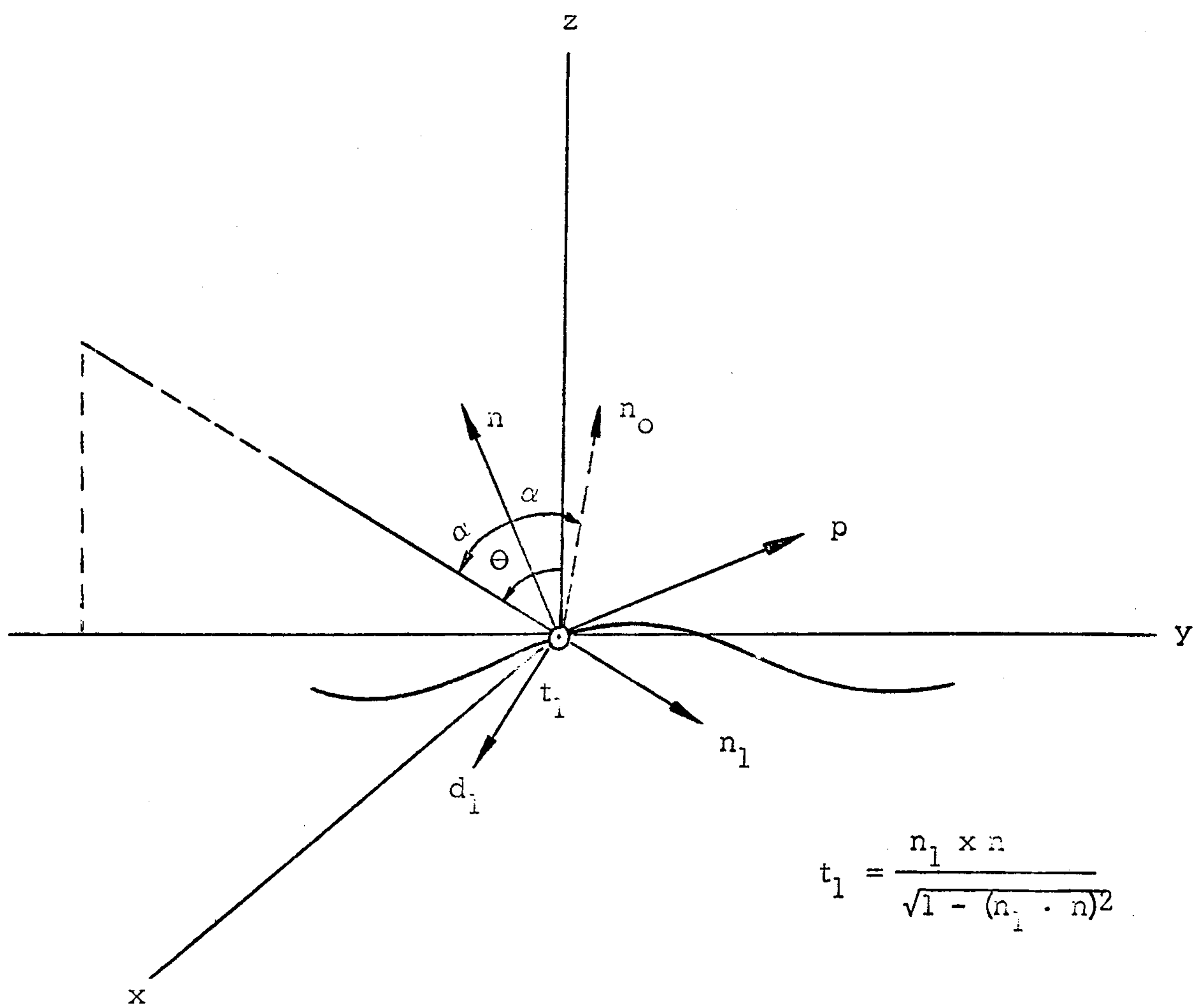


Figure 1 Scattering Diagram

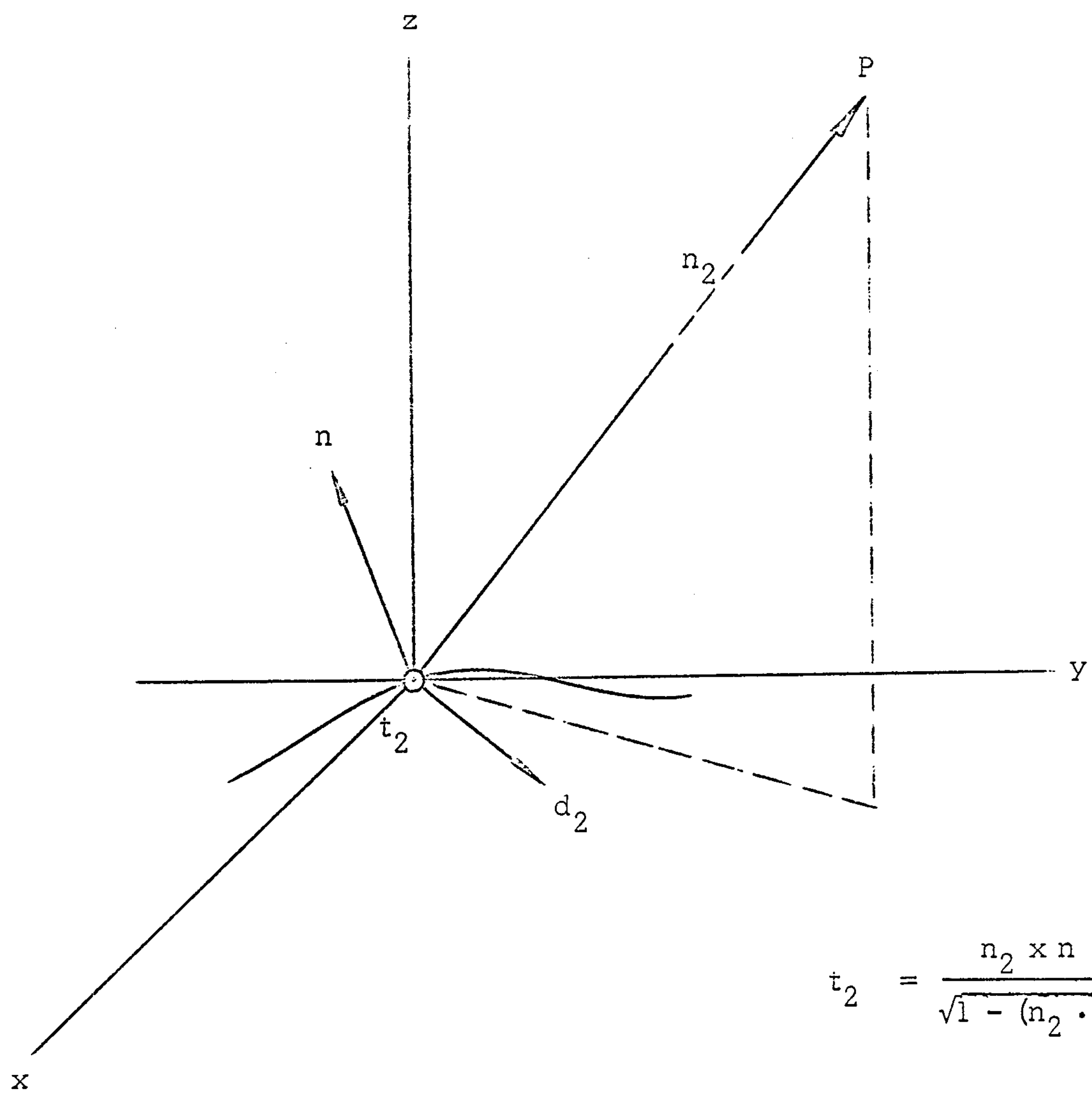


$$t_1 = \frac{n_1 \times n}{\sqrt{1 - (n_1 \cdot n)^2}}$$

$$p = n \times t_1$$

$$d_1 = n_1 \times t_1$$

Figure 2 Local Coordinate System 1



$$t_2 = \frac{n_2 \times n}{\sqrt{1 - (n_2 \cdot n)^2}}$$

$$d_2 = n_2 \times t_2$$

Figure 3 Local Coordinate System 2

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